26-5

THEORETICAL COMPARISON OF SIX-PORT REFLECTOMETER JUNCTION DESIGNS

E J Griffin and T E Hodgetts

Royal Signals and Radar Establishment
St Andrews Road, Malvern, Worcestershire WR14 3PS, England

ABSTRACT

This paper derives numerical procedures for comparing different theoretical designs of six-port junctions for measuring the voltage reflection coefficient Γ of passive loads ($|\Gamma| \leqslant 1$). It shows that the maximum uncertainty of measuring any $|\Gamma| \leqslant 1$ can be minimised, by a suitable choice of components, for each of three published designs.

INTRODUCTION

Since Hoer and Engen first described the use of a six-port reflectometer for measuring Γ (1-3), a number of different designs of junction for this type of instrument have been described. This range of different designs confronts the potential user with the question: "Can their likely performance be compared?" Given a maximum permitted power P_D at any detector and an equivalent noise power P_N at each detector, we derive as criteria for this comparison:

(i) the maximum uncertainty U_{MAX} for measuring any $|\Gamma| \leqslant 1$ when the reference detector absorbs P_D, and
(ii) the maximum power P_{MAX} that can be incident on the junction without the power at any detector exceeding P_D. We then show that U_{MAX} can be minimised for each of three published designs by a suitable choice of components.

MAXIMUM UNCERTAINTY U_{MAX}

It is well known that the ratios of power absorbed by three of the detectors P_K (K = 1,2,3) of a six-port reflectometer to that absorbed by the fourth, reference, detector P_R are related to Γ by:

$$P_K/P_R = |(d_K\Gamma+e_K)/(c\Gamma+1)|^2 \qquad (K = 1,2,3)$$

where c, d_K, e_K are dimensionless numbers describing the instrument in terms of calibration standards.

Because the instrument relies on calibration, it is sufficient and usual for design purposes to assume the reference detector to be isolated from the wave reflected by the device under test, which enables c to be equated to zero. This allows the instrument to be described by:

$$R_K^2 = D_K^2(P_K/P_R) = |\Gamma-f_K|^2 \qquad (1)$$

where $D_K = |d_K|^{-1}$ and $f_K = -(e_K/d_K)$.

Equations of the form of (1) are presented later for three different designs of six-port junctions. These equations each represent in the complex Γ plane a circle of radius R_K centred at f_K and Γ is found from their intersection.

Noise present in the output of each detector will cause uncertainty in determining each R_K and this can be represented by a rectangular probability distribution of R_K between limits of $\pm \Delta R_K$, caused by an equivalent noise power P_N for each detector. Then, from (1):

$$R_K \pm \Delta R_K = D_K((P_K \pm P_N)/(P_R \mp P_N))^{\frac{1}{2}} \qquad (2)$$

Assuming that $P_N \ll P_K$ and $P_N \ll P_R$ then

$$\frac{\Delta R_K}{R_K} \simeq \pm \tfrac{1}{2}\left(\frac{1}{P_R} + \frac{1}{P_K}\right)P_N \qquad (3)$$

The minimum of this fractional uncertainty ($\Delta R_K/R_K$) occurs when $P_K = P_R = P_D$, but this cannot be achieved for all Γ so we choose to try operation with the reference detector absorbing the maximum power P_D. Then, equation (2) becomes:

$$\frac{\Delta R_K}{R_K} \simeq \pm \tfrac{1}{2}\left(1+\frac{P_D}{P_K}\right)\frac{P_N}{P_D} \qquad (4)$$

and should the design be such that $P_R < P_D$ then $\Delta R_K/R_K$ can be scaled by the factor P_R/P_D.

In the region of the intersection of R_1, R_2 and R_3 (from which Γ is calculated), each pair of limits ($\Delta R_1,\Delta R_2$), ($\Delta R_2,\Delta R_3$), ($\Delta R_3,\Delta R_1$) defines a curvilinear parallelogram within which Γ lies, as illustrated in Figure 1.

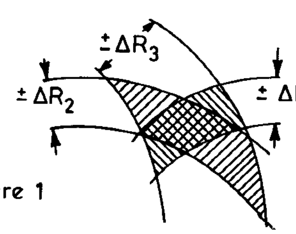


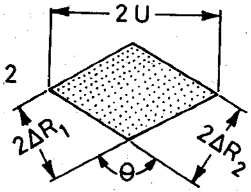
Figure 1

0149-645X/84/0000-0187 $01.00 © 1984 IEEE

Because $\pm\ \Delta R_K$ define limits of a rectangular probability distribution of R_K, it is certain that Γ lies within the smallest of these three parallelograms, as shown by the cross-hatched area of Figure 1. Now for the parallelograms of interest, $\Delta R_K \ll R_K$ because the only purpose of the smallest of the three R_K is to resolve which of two intersections of the two largest R_K relates to Γ. This allows each curvilinear parallelogram to be approximated by a rectilinear parallelogram, as shown in Figure 2. The cosine law can therefore be used for calculating the maximum diagonal 2U from:

$$U = ((\Delta R_1)^2+(\Delta R_2)^2+2(\Delta R_1)(\Delta R_2)|\cos\theta|)^{\frac{1}{2}}/\sin\theta \qquad (5)$$

Figure 2



Equations (1), (4) and (5) allow the limits of $\pm U$ to be estimated for any Γ as the smallest of the three semi-diagonal lengths U obtained by considering the three ΔR_K in pairs.

Relating the limits of $\pm U$ so calculated to measurement of Γ relies on the fact that the angular orientation of the maximum diagonal of Figure 2, with respect to the axes of the Γ plane, has no significance until the reflectometer has been calibrated with external standards. Thus the range from $-U$ to $+U$ can only be regarded as defining the diameter of a circle of confusion (to borrow a term from optics) within which it is certain that Γ lies. Hence the estimated uncertainty in measuring magnitude $|\Gamma|$ is $\pm U$ and in measuring phase angle $\angle\Gamma$ it is $\pm\arctan(U/|\Gamma|)$. Finally, we can compute each U for a net of Γ covering the $|\Gamma| = 1$ radius circle to select the maximum U_{MAX} in measuring any $|\Gamma| \leqslant 1$. The estimated maximum uncertainties U_{MAX} provided later for different junctions were obtained by using this procedure with 321 different Γ evenly spaced over the $|\Gamma| = 1$ radius circle.

MAXIMUM POWER P_{MAX}

We have postulated that the reference detector (i) is isolated from the reflected wave and (ii) absorbs the maximum permitted detector power P_D. The net power supplied to the instrument from a matched source with available power output P_o will vary with Γ but a consequence of (i) is that P_R is a constant fraction F of P_o, irrespective of Γ, so that:

$$P_R = FP_o \qquad (6)$$

A consequence of (ii) is that it is necessary to check whether the condition $P_R = P_D$ to maximise resolution can be met and, if not, to scale the computed U_{MAX} by the factor P_R/P_D. But for each K, the maximum of P_K can be calculated from equation (1) and for one K (say K = n) this P_{nMAX} will be the greatest of the three. We require that $P_{nMAX} \ngtr P_D$ for which, from (1):

$$\frac{P_D}{P_R} = \frac{(1+|f_n|)^2}{D_n^2} \qquad (7)$$

Ideally, then, we require that $(1+|f_n|)^2/D_n^2 = 1$ and, if not, then the computed U_{MAX} must be scaled by P_R/P_D given by equation (7). Finally, the maximum power that can be incident on the junction to minimise U_{MAX} is, from equations (6) and (7):

$$P_o = \frac{D_n^2\ P_D}{F(1+|f_n|)^2} \qquad (8)$$

We present the results of applying equations (6) to (8) for different six-port junctions below.

EXPLANATION OF TABULATED RESULTS

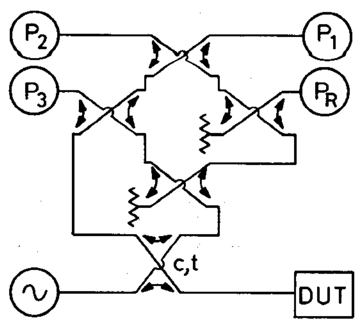
A comparison is presented below of three designs of six-port junctions (4-7) using the procedures derived, together with a diagram of each. Each junction comprises a number of 90° hybrids and one directional coupler of coupling factor 20 $\log_{10}$ (1/c) having a voltage transmission coefficient t (so that $|t|^2+|jc|^2 = 1$). The diagram given below for the design of reference (4) assumes construction from Lange couplers, while the remainder assume conventional waveguide components; on each diagram the coupled paths are denoted by arrows (thus ↔). Below each diagram are provided the values of D_K^2 and f_K of equation (1) appropriate to the design. Below these are tabulated:

(a) the coupling factor $C = 20 \log_{10} (1/c)$ dB,
(b) the ratio P_D/P_R,
(c) the maximum power P_{MAX} (in terms of P_D),
(d) U_{MAX} for all $|\Gamma| \leqslant 1$ when P_{MAX} is incident on the junction as a multiplier of P_D/P_N (the maximum detector signal-to-noise ratio).

The minimum U_{MAX} and the coupling factor C giving this minimum are starred (thus*).

DESIGN OF REFERENCE (4)

Diagram:



Coefficients for equation (1):

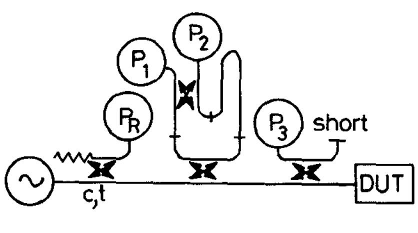
K	D_K^2	f_K
1	$1/c^2$	$-\frac{1}{\sqrt{2}c}(1+j)$
2	$1/c^2$	$-\frac{1}{\sqrt{2}c}(1-j)$
3	$1/2c^2$	$\frac{1}{\sqrt{2}c} + j0$

Computed values:

C dB (a)	P_D/P_R (b)	P_{MAX} (c)	$U_{MAX}(P_D/P_N)$ (d)
3	4.01	2.00 P_D	14.01
6	2.92	1.83 P_D	12.19
10*	5.81	2.12 P_D	12.17*
20	15.39	3.10 P_D	20.05

DESIGN OF REFERENCES (5,6)

Diagram:



Coefficients for equation (1):

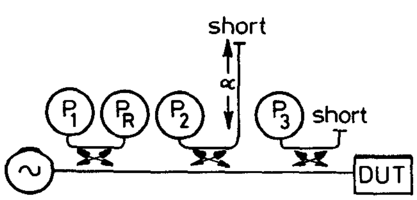
K	D_K^2	f_K
1	$32\ c^2/t^2$	$-(1+j2\sqrt{2})$
2	$32\ c^2/t^2$	$-(1-j2\sqrt{2})$
3	$8\ c^2/t^2$	$1+j0$

Computed values:

C dB (a)	P_D/P_R (b)	P_{MAX} (c)	$U_{MAX}(P_D/P_N)$ (d)
3.0	1.00	2.0 P_D	14.13
4.8*	1.00	3.0 P_D	8.30*
6.0	1.49	4.0 P_D	9.92
10.0	4.50	10.0 P_D	18.69

DESIGN OF REFERENCE (7)

Diagram:



Coefficients for equation (1):

K	D_K^2	f_K
1	$16/t^2$	$-(1-2(\cos 2\alpha - j\sin 2\alpha))$
2	$16\ c^2/t^2$	$-(1+2(\cos 2\alpha - j\sin 2\alpha))$
3	$8\ c^2/t^2$	$1+j0$

Computed values for largest U_{MAX} (when $\alpha = 60^{\circ}$):

C dB (a)	P_D/P_R (b)	P_{MAX} (c)	$U_{MAX}(P_D/P_N)$ (d)
3.0	1.0	2.0	13.80
3.4*	1.0	2.2	12.06*
6.0	2.48	4.0	21.52
10.0	7.48	10.0	53.79

DISCUSSION

The ratio P_D/P_N represents the maximum possible signal-to-noise ratio at any detector and the tabulated $U_{MAX}(P_D/P_N)$ show the extent to which this ratio is degraded by each junction, even when P_{MAX} is incident on the junction. The starred values in the tables show that the worst case uncertainty in measuring any $|\Gamma| \leqslant 1$ can be minimised for each design by a suitable choice of coupling factor and that the design of references (5,6) offers the smallest uncertainty of the three considered, albeit at the expense of more RF power.

CONCLUSION

We have derived numerical procedures for comparing theoretical designs of six-port junctions and have shown that the worst case uncertainty of measurements may be minimised by design.

REFERENCES

(1) C A Hoer, IEEE Trans, IM-21, 466-470, Nov 1972.

(2) G F Engen, ibid, 470-474.

(3) G F Engen, IEEE Trans, MTT-25, 1075-1080, Dec 1977.

(4) G F Engen, ibid, 1080-1083.

(5) UK Patent Application 8311170, May 1983.

(6) E J Griffin, G J Slack, L D Hill, Electron Lett, 19, 921-922, Oct 1983.

(7) E J Griffin, Electron Lett, 18, 491-493, Jun 1982.